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How do Students Think in Translating Verbal Representation to Graphics?

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Abstract. This research is based on the variety of students' work in completing mathematical translations, especially from verbal representations to graphs. This study aimed to analyze the path of students' mathematical translation thinking from verbal representations to graphs. Thirty-two students were involved in completing the mathematical translation task, and four students were selected as research subjects. The supporting instruments in this research are in the form of mathematical translation tasks and interview guidelines. The data analysis step begins by grouping the students' work and making a transcript of the interview results. Next, the researcher explored and coded the students' work, found differences in the mathematical translational thinking path, explained the mathematical translation process for each path, reported the findings, interpreted the findings, and validated the research results by triangulating data sources. This study resulted in two types of students' mathematical translational thinking paths, namely the complete and incomplete construction translational thinking path. The difference between these two paths lies in the completeness of cognitive activity in each step of mathematical translation. The results of this study are used as considerations in designing meaningful mathematics learning activities.

Keywords: Graphics Representation; Mathematical Translation; Thinking Path; Verbal Representation

Abstrak. Penelitian ini dilatarbelakangi oleh beragamnya hasil kerja siswa dalam menyelesaikan translasi matematis, terutama dari representasi verbal ke grafik. Tujuan dari penelitian ini adalah untuk menganalisis jalur berpikir translasi matematis siswa dari representasi verbal ke grafik. Tiga puluh dua siswa terlibat dalam menyelesaikan tugas translasi matematis dan 4 siswa dipilih sebagai subjek penelitian. Instrumen pendukung dalam penelitian ini berupa tugas translasi matematis dan pedoman wawancara. Langkah analisis data dimulai dengan mengelompokkan hasil kerja siswa dan membuat transkrip hasil wawancara. Selanjutnya peneliti menggali dan mengkodekan hasil karya siswa, menemukan perbedaan jalur berpikir translasi matematis, menjelaskan proses translasi matematis untuk setiap jalur, melaporkan temuan, menginterpretasikan temuan, dan memvalidasi hasil penelitian dengan triangulasi sumber data. Penelitian ini menghasilkan dua jenis jalur berpikir translasi matematis siswa, yaitu jalur berpikir translasi konstruksi lengkap dan tidak lengkap. Perbedaan antara kedua jalur ini terletak pada kelengkapan aktivitas kognitif pada setiap langkah translasi matematis. Hasil penelitian ini digunakan sebagai bahan pertimbangan dalam merancang kegiatan pembelajaran matematika yang bermakna.

Kata kunci: Alur Berpikir; Representasi Grafik; Representasi Verbal; Translasi Matematis

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INTRODUCTION

Mathematical problems are often presented in various forms of mathematical representation such as symbols, verbal, graphs, pictures and tables. Solving these problems requires complex cognitive activities, such as identifying specific transformations from one form of mathematical representation to another (Leinhardt et al., 1990), interpreting ideas, concepts and mathematical properties of two or more representational structures (Roth & Bowen, 2001) and create a network of ideas or constructions from one form of representation to another (Adu-Gyamfi et al., 2012). Research result (Lesh & Behr, 1987) summarizes all these cognitive activities into mathematical translational thinking activities. Students' translational thinking skills become a catalyst in achieving deep mathematical understanding (Adu-Gyamfi et al., 2019; Association of Mathematics Teacher Educators, 2017) and strong mathematical connections (Afriyani, 2018).

Several previous studies on mathematical translational thinking have found: that (1) students' mathematical translation processes vary and are determined by their level of ability (Bossé et al., 2014), the type of mathematical understanding (Afriyani et al., 2018) and the type of mathematical connection (Adu-Gyamfi et al., 2017), and (2) mathematical translational thinking is a cognitive strategy used by students or students in completing multiple tasks of mathematical representation (Afriyani et al., 2018), word problems (Muttaqien, 2016) and constructing graphs of compositional functions (Afriyani & Yuberta, 2019). Aside from finding revealed by Afriyani & Yuberta (2019) is that the mathematical translation strategy displayed by students has variations in conceptual content transfer in the step of constructing the target representation. This variation is due to differences in the type of mapping carried out at the preliminary coordination stage. The diversity of mathematical translation processes raised by students causes the diversity of students' mathematical translational thinking lines. However, previous research has not investigated the path of mathematical translational thinking.

Research results revealed that the students' mathematical translation process is a construction path of mental structures and mechanisms (Afriyani, 2018). Furthermore, Afriyani (2018) explains that the students' mathematical translation process begins with interiorization, coordination, reversal, encapsulation and thematization. In each mental mechanism, a mental structure will be formed in actions, processes, objects and schemas. The completeness of the mental structure construction formed in completing the mathematical translation task depends on the completeness of the mental mechanisms carried out by the students (Arnon et al., 2014). Therefore, it is necessary to analyze students' mathematical translational thinking path, and previous research has not revealed this. In this case, This study reveals how students' thinking in completing mathematical translation tasks refers to the mathematical translation steps expressed by (Bossé et al., 2014). This study also deepens our understanding of the emergence and types of constructions

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in the mathematical translation step. Finally, the results of this study can detect the weaknesses of students' mathematical translational thinking processes and provide recommendations for learning mathematics to cover these weaknesses.

METHOD

Mathematical translational thinking path data were obtained through tests and interview guidelines considered valid by three mathematicians. The test asks students to graph a function of two given linear inequalities. The task or mathematical translation test (TTM) used to explore students' mathematical translation from verbal to graphical representation is as follows.

> "*To increase income, a housewife every day produces two types of cakes to sell. Each cookie has a capital of Rp. 1,000.00 with a profit of Rp.800,00 while each wet cake has a capital of Rp. 1.500.00 with a profit of Rp.900.00. If the available capital per day is Rp. 500.00 and can only produce 400 cakes at most, what is the biggest profit that the housewife can get?*"

This mathematical translation task was given to thirty-three students. Then four students were randomly selected from the correct translation to be interviewed to explore in-depth information and strengthen the answers to the previous test results. The complete construction thinking path processes and incomplete construction thinking process were explored from two subjects.

Data were analyzed in eight steps. **First**, classify student work. At this stage, the researcher groups the correct answers and then sorts out the students who are right and fail to translate between representations. **Second,** make a transcription of the interviews' results, where the interview activities' implementation was recorded, and then the recordings were presented in written form. **Third,** researchers explore and code students' work. At this stage, the researchers to find and find similarities and differences in the patterns of students in giving answers, then made a code for each translation step carried out by students. Then a code is made for each translation step carried out by students. The code makes it easier to get students' translational thinking path. The students' cognitive activity codes are shown in Table 1. **Fourth,** find differences in the path of mathematical translation thinking. Here, the researcher describes the translation steps performed by each subject. **Fifth,** explain the process of mathematical translation of each plot. The researcher describes what students do at this stage who have complete and incomplete construction thinking lines. **Sixth,** reporting the findings. **Seventh**, interpreting the findings. **Eighth**, validating the study results by triangulating data sources, namely the researcher re-matching the results of student work with the results of interviews.

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Table 1. Description of Students' Mathematical Translation Steps

Steps	Operational description	Code
Unpacking the source	Reading questions	U_1
	Reveal the price of pastries	U_2
	Reveal the price of wet cake	U_3
	Reveal the advantages of each cake	U_4
	Disclosing available capital	U_5
	Reveal the many cakes	U_6
	Expressing what is asked is the maximum profit	U_7
Preliminary coordination	Identify the relationship between the number of cookies, cakes and modal	I_1
	Identify the number of cakes to be produced, i.e. no more than 400 pieces	I_2
Constructing the target representation	Assume a cookie with a variable	C_1
	Assume a wet cake with a variable	C_2
	Finding the inequality $1000x + 1500y \leq 500000$	C_3
	Finding the inequality $x+y \leq 400$	C_4
	Finding the inequality $x \geq 0$	C_5
	Finding the inequality $y \geq 0$	C_6
	Determine the corner points A, B and C	C_7
	Substitute the values of A,B,C in the function $z = 800x + 900y$	C_8
	Find the maximum value	C_9
Determining equality	Explain the meaning of the relationship between inequalities and function graphs	D_1

RESULTS AND DISCUSSION

The results showed two forms of thinking path found in students, namely complete construction thinking path and incomplete construction thinking path. The complete construction thinking path is a thought process carried out by students in completing translation tasks using concepts that have been studied previously and understanding these concepts so that every step they take is structured and related. The path of incomplete construction thinking is a thinking process carried out by students using concepts, but the concepts used are not wholly complete or do not understand the concept, so in solving problems, the steps used are not appropriate.

Subjects included in the complete construction thinking path are 12 subjects, 20 subjects are included in the incomplete construction thinking path, and every two subjects will be presented. Subject 1 (S1) and subject 2 (S2) for the complete construction translational thinking path and subject 3 (S3) and subject 4 (S4) for the incomplete construction translational thinking path.

Complete Construction Translational Thinking

The complete construction thinking path is a thinking process carried out by students in completing mathematical translation tasks with concepts that have been studied previously and understanding these concepts so that every step they take is structured and related. Exposure to the thinking structure of the subject which made a mistake was represented by two subjects, namely S1 and S2. The translation process from verbal representation to symbolic representation carried out by S1 and S2 has the same line of thinking and is depicted in Figure 1.

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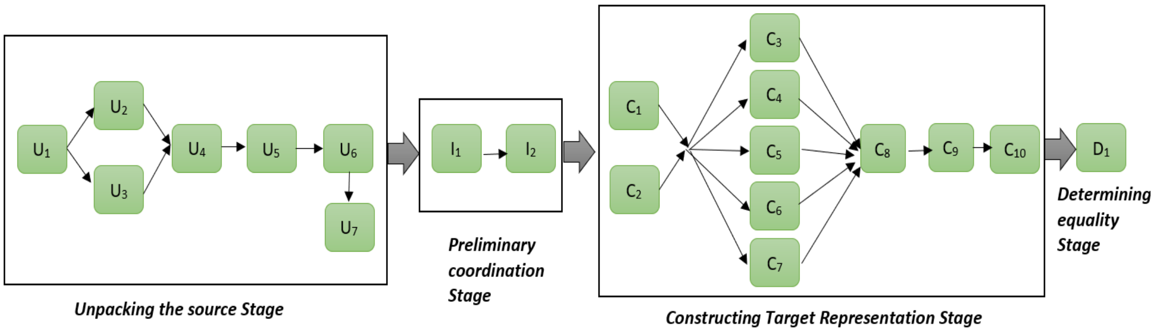

Figure 1. Complete construction translational thinking path

The codes used, such as U1, U2, and others, refer to Table 1, while the arrows indicate the subject's thought process direction. Subjects who fall into the complete construction translational thinking flow bring up all cognitive activities in each translation step from verbal to symbolic representations. The description of the S1 and S2 thinking path when completing the translation task from verbal representation to graphic representation is as follows.

Unpacking the source Stage

Subject 1 (S1) and Subject 2 (S2) revealed that the information that was known was that the pastry capital was Rp. 1,000,- with a profit of Rp. 800,- and wet cakes of Rp. 1,500,- with a profit of Rp. 900,- with an existing capital of Rp. 500,000,- with a maximum of 400 pieces, then the decisive step is the problem's primary and objective function. It can be seen that S1 and S2 identify the information they know from the questions, namely the price of pastries and wet cakes as well as the benefits of each cake and the available capital and the number of cakes. The presentation of information on the questions made by S1 and S2 is in the form of tables, as can be seen in Figure 2 and Figure 3.

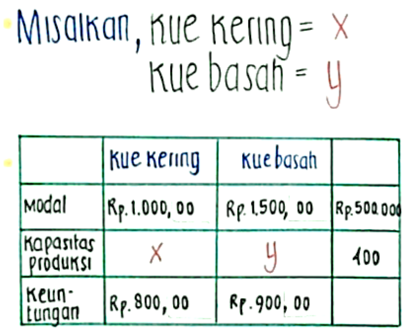

Figure 2. The Answer of S1

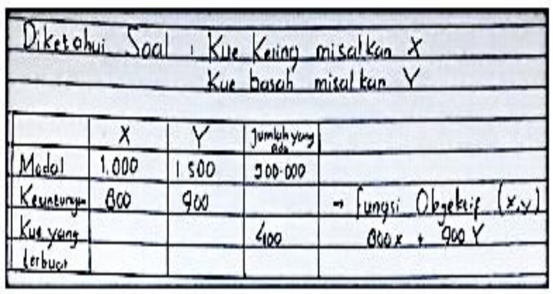

Figure 3. The Answer of S2

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Preliminary coordination Stage

At this stage, S1 and S2 identify essential things in the problem and relate them, such as the relationship between the price of cookies, cakes and capital. S1 and S2 also know many cakes that homemakers will produce. Next, S1 and S2 think about inequality involving two variables.

Constructing Target Representation Stage

In constructing the target representation, S1 and S2 assume cookies with X and cookies with Y. Next, S1 and S2 create two inequalities containing variables X and Y, as follows: 1000x+1500y 500,000 and x+y 400, $x \geq 0, y \geq 0$. From the interview results, the S1 and S2 thought processes in making the two inequalities were obtained. Next, S1 and S2 draw an inequality graph to find the solution area (DHP) by finding some coordinates of the critical points and testing them with point O (0,0) so that we get three corner points on the DHP where the coordinates of the intersection point are obtained by the elimination method. The graphs made by S1 and S2 are shown in Figure 4 and Figure 5, which show that the yellow and white areas are DHP, which are the areas most frequently affected by shading.

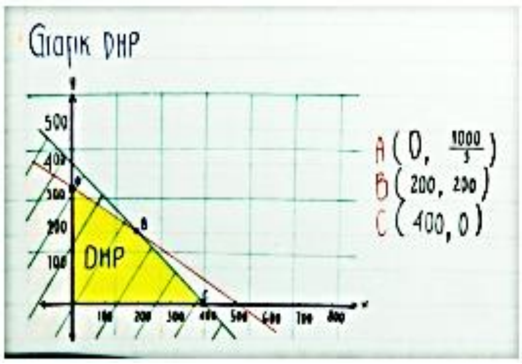

Figure 4. Graph of inequality by S1

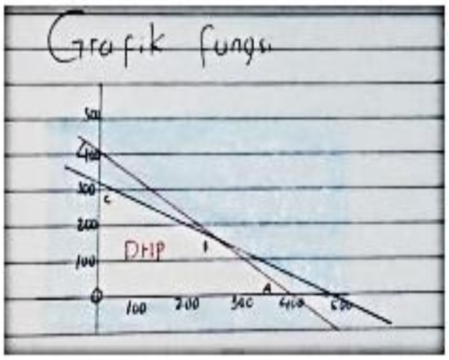

Figure 5. Graph of inequality by S2

The corner points obtained, as shown in Figure 4 and Figure 5, are substituted into the objective function z = 800x + 900y so that the maximum point is obtained. Thus S2 finds the maximum value after substituting corner points in the objective function. It follows the exposure of S2 in answering the test. The results of the S2 paraphrase are shown in Figure 6.

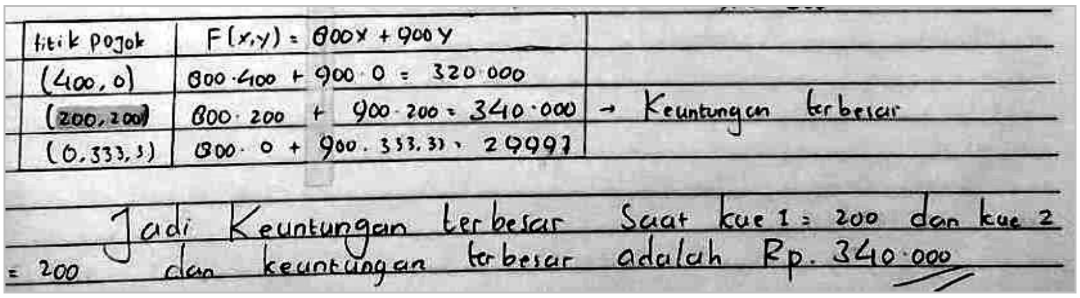

Figure 6. The Answer of S2

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Determining equality Stage

This stage of determining equality asks S1 and S2 to state the meaning or relationship between inequalities found at the beginning with the form of the graph they made, where students answer that the meaning of the graph is the same as the inequality because the graph is a modified form of inequality. So the conclusion found by S1 and S2 is that the profit earned by the housewife is Rp. 340,000.

Incomplete Construction Translational Thinking

Incomplete construction thinking is less able to state what it makes correctly, incomplete in making plans for completion and less able to correct or check the correctness of the answer. The path of incomplete construction thinking is a thinking process carried out by students using concepts, but the concepts used are not entirely complete or do not understand the concept, so in solving problems, the steps used are not appropriate. The thinking structure of the subject who carries out the line of thinking is represented by two subjects, namely S3 and S4. The translation process from verbal representation to symbolic representation carried out by S1 and S2 has the same line of thinking and is depicted in Figure 7.

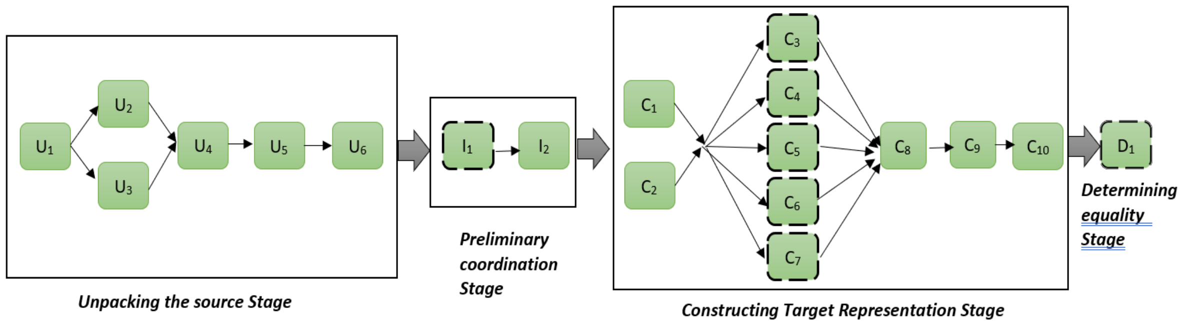

Figure 7. Incomplete construction translation thinking path

This section describes the data on the translation process carried out by S3 and S4 when completing the translation task from verbal representation to graphic representation. In Figure 7, the dotted line box shows that cognitive activity in verbal to symbolic translation is incomplete or incomplete. Students in this category do not express what is asked as the maximum profit at the unpacking, the source stage.

Unpacking the source Stage

After reading the questions, S3 and S4 identify essential things in the questions. Then, make what he knows, namely the capital of pastries and wet cakes, the profit from each of these cakes, the capital of the whole cake and the number of cakes produced. The information that S3 found in the question was that the capital of cookies was Rp. 1,000 with a profit of Rp. 800 and the price of wet

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cakes was Rp. 1,500 with a profit of Rp. 900, the capital of the cake was Rp. 500,000, and the number of productions was 400. Here is the answer he wrote, as shown in Figure 8.

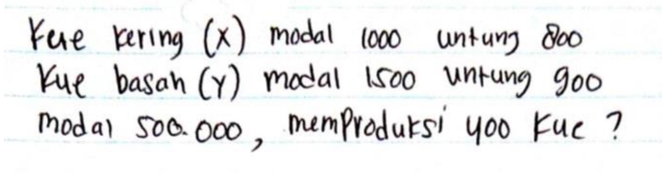

Figure 8. Answers for unpacking sources

After knowing the essential things from the questions, S3 obtains the information that he must determine from the questions and steps to solve the problems.

Preliminary coordination Stage

The preliminary coordination stage requires S3 and S4 students to see and interpret the relationship of each information disclosed at the stage of disassembling the source. S3 and S4 students were asked to make a relationship between the capital of each type of cake with the available capital, but S3 and S4 students were not sure, and we could not describe it clearly. It is different from the above conditions. S3 and S4 students can see the relationship between the number of each type of cake and the maximum amount that must be adequately produced.

Constructing Target Representation Stage

At the stage of constructing this target, S3 and S4 students can assume each type of cake with a variable and determine the inequality's shape, but it is incomplete and cannot explain why using the same small sign in inequality makes. The above statement is supported by the results of S3 and S4 work, as shown in Figure 9 and Figure 10.

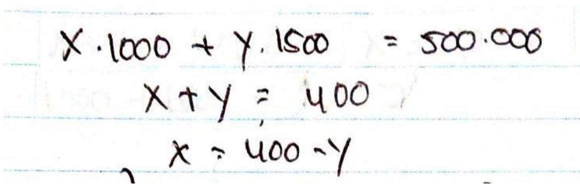

Figure 9. The Answer of S3

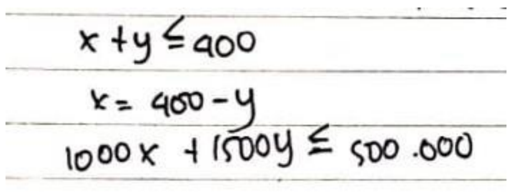

Figure 10. The Answer of S4

The two equations obtained by S3 and S4 are then substituted for the x and y values. After the coordinates of the point are obtained, the value is multiplied by the profit of each type of cake, as stated in Figure 11 and Figure 12.

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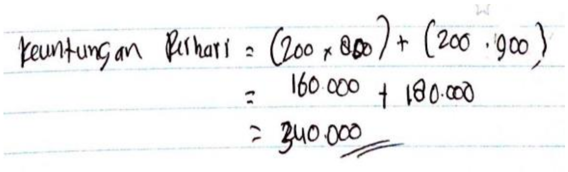

Figure 11. The Answer of S3 on finding profit

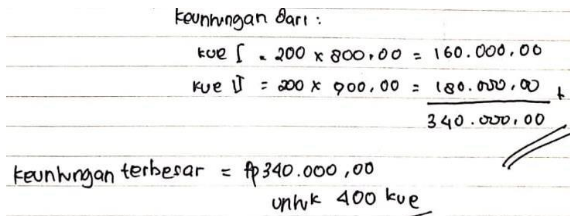

Figure 12. The Answer of S4 on finding profit

Determining equality Stage

After that, in the stage of determining the equivalence of S3, it was asked about the suitability of the answers obtained with the purpose of the question. S3 stated that the values of x and y found were the same as the maximum number that the housewife was able to produce, which was 400 pieces so that with that, the housewife earned a profit of Rp. 340,000.

The mathematical translation path of complete and incomplete constructions found in this study shows differences in performance in completing translations. One of them is caused by differences in mathematical ability (Adu-Gyamfi et al., 2019) and students' mathematical understanding (Afriyani & Pramita, 2021; Rau & Matthews, 2017). The path of thinking that is passed by students who have complete and incomplete construction starts from dismantling the source, carrying out initial coordination, constructing targets and determining targets. However, there are differences between the completeness of cognitive activity at each translational step. The difference between the two paths can be seen in Figure 13 and Figure 14. This finding is similar to the results of Hamda's research (Hamda, 2020).

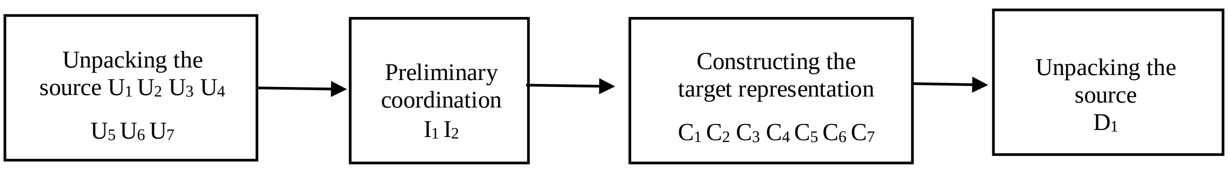

Figure 13. Complete construction translation thinking path

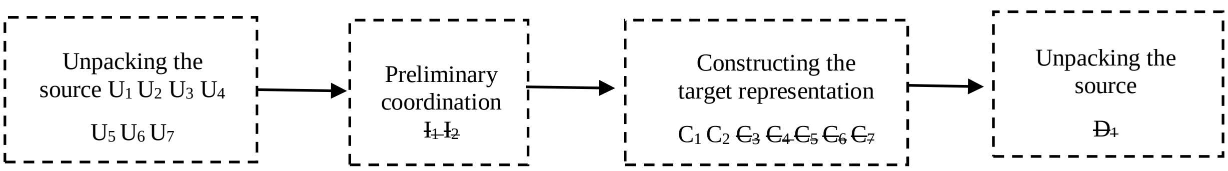

Figure 14. Incomplete construction translation thinking path

Students who have incomplete translational thinking lines are caused by incomplete cognitive activity in several translation steps. The dotted line represents the incompleteness in Figure 14. The incompleteness begins at the initial coordination step, namely the inability to identify the relationship between some of the information obtained from dismantling the source and how that information can be used to solve the problem. In this case, Afriyani & Pramita (2021) mentions that this occurs when students fail to establish a connection between the structure of the

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representation of the source and the representation of the target. The incomplete construction in the initial coordination step causes students to experience obstacles in the target construction step (Abdullah et al., 2015; Zulianto & Budiato, 2020). In addition, students who have incomplete construction flow cannot explain the concepts used and the correctness of the answers made. These conditions are included in indicators of incomplete construction, namely being unable to explain in their language, not understanding concepts and mixing general concepts with intuition (Lilia, 2017).

CONCLUSION

This study found two forms of thinking path for translation of verbal representations to graphics: complete construction translational thinking path and incomplete construction translational thinking path. The difference between the two lines of thinking can be seen from the completeness of cognitive activity in each step of mathematical translation. Students apply the concept correctly and understand the concept in the complete construction translational thinking path. Students apply the concept in the incomplete construction path, but the concept used is not entirely complete or does not understand the concept. This research is only limited to the form of translation of verbal representations to graphs. For that, it is still open to be studied in further research.

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