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Unveiling Students' Refractive Thinking Process Using a Single Strategy in Decision Making

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Abstract. The thinking process characterized by reflective thinking towards critical thinking to produce a decision is called the refractive thinking process. However, how the refractive thinking process occurs in the subject needs to be revealed. This study aims to reveal the refractive thinking process in subjects who use a single strategy in decision making. This study is a qualitative approach involving 25 mathematics education students from two universities in Malang, East Java, Indonesia. Each student was given a test on the problem of ordering cities based on the level of dependence on the central government from lowest to highest. The decision to order cities was based on local revenues data that varied between cities and unstable from year to year. In this study, the researcher only analyzed the refractive thinking process of one subject who used a single strategy in decision making. The subject was asked to think aloud. The results showed that students who carried out the refractive thinking process with a single strategy only needed one alternative solution to make a decision, namely by summing up the contributions of each city per year. The subject only identified the comparison of the amount and increase in the contribution of each city as a consideration for making a decision.

Keywords: Critical Thinking; Decision Making; Reflective Thinking; Refractive Thinking; Think Aloud

Abstrak. Proses berpikir yang ditandai oleh proses berpikir reflektif menuju berpikir kritis hingga menghasilkan suatu keputusan disebut proses berpikir refraktif. Namun, bagaimana proses berpikir refraktif terjadi pada subjek perlu diungkap. Penelitian ini bertujuan untuk mengungkap proses berpikir refraktif pada subjek yang menggunakan strategi tunggal dalam pengambilan keputusan. Penelitian ini merupakan pendekatan kualitatif yang melibatkan 25 mahasiswa pendidikan matematika dari dua universitas di Malang, Jawa Timur, Indonesia. Setiap mahasiswa diberikan tes tentang masalah pengurutan kota berdasarkan tingkat ketergantungan terhadap pemerintah pusat dari terendah hingga tertinggi. Keputusan pengurutan kota didasarkan pada data pendapatan asli daerah yang bervariasi antarkota dan tidak stabil dari tahun ke tahun. Dalam penelitian ini, peneliti hanya menganalisis proses berpikir refraktif satu subjek yang menggunakan strategi tunggal dalam pengambilan keputusan. Subjek diminta untuk berpikir nyaring (*think aloud*). Hasil penelitian menunjukkan bahwa mahasiswa yang melakukan proses berpikir refraktif dengan strategi tunggal hanya membutuhkan satu alternatif penyelesaian untuk mengambil keputusan, yaitu dengan menjumlahkan kontribusi setiap kota per tahun. Subjek hanya mengidentifikasi perbandingan jumlah dan peningkatan kontribusi setiap kota sebagai pertimbangan untuk mengambil keputusan.

Kata kunci: Berpikir Kritis; Berpikir Nyaring; Berpikir Reflektif; Berpikir Refraktif; Pengambilan Keputusan

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INTRODUCTION

Research on data processing has gain much of attention from researchers. For instance, the mistake make graph because incorrect data processing (Curcio, 2001; Van de Walle et al., 2006; Garfield & Gal, 1999; Harper, 2004) and the difficulties of students collect data and process data so that affects in making decision (UNCMSE, 1997; McClain & Cobb, 2000; Greer, 2000; Manchester, 2002). To avoid occurrence of irregularities in problem solution, Doerr & English (2003) developed stage of completion include: interpretation, description, conjecture, explanation, and evaluation. In the view of Jansen & Spitzer (2009), descriptions and interpretations developed by Doerr & English (2003) is stage of reflective thinking. Plymouth University (2010) and Facione (2013) explain that conjecture, explanation, and evaluation is stage of critical thinking. This indicates that stage of completion developed by Doerr & English (2003) consist of two processes of thinking, namely reflective thinking and critical thinking. According to Pagano and Roselle (2009) and Medeni & Medeni (2012) stated that the process produces knowledge through reflection and critical thinking is refraction. Therefore thinking is signed with reflective thinking continued critical thinking till produce decision called refractive thinking. This indicates that an important component of refractive thinking is reflective thinking, critical thinking and decision (product).

Pagano & Roselle (2006, 2009) defines that refraction is transformative knowledge that occurs the which validates the use of critical analysis and problem solving providing interpretation and conclusions of important issues and situations considering the course content and context. Knowledge transformative in this case is ability of person solved problems through some alternative solution. The purpose of refraction is process decision-making by considering some possible alternative solution (Pagano & Roselle, 2006; Pagano & Roselle, 2009). This shows that refraction is focusing of information since there are some alternative solution obtained when reflection and critically analysis as consideration to establish a decision. Be related with refraction, Medeni & Medeni (2012) defines that refraction is new knowledge acquisition from critical thinking of reflection. This shows that the refraction thinking is the process of acquiring new knowledge (decision) result from reflection and critical thinking. Therefore refractive thinking this study is process of decision making through reflective thinking continued with critical thinking.

Reflective Thinking is thinking process important in construct of knowledge. Reflective thinking is initiated by the perception of something troubling or promising, and it is determined by the production of changes one finds on the whole satisfactory or by the discovery of new features which give the situation new meaning and change the nature of questions to be explored (Schon, 1991). This shows that, reflective thinking signed with difficulty (trouble) experienced by person so that he doing continuously behavior changes. Behavior changes are the process of investigated with explore information on the problem. Investigations done to resolve the situation of

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uncertainty, instability, uniqueness, and conflict so that as provide answers the questions. Based on the above definition reflective thinking, implicitly there are some components of reflective thinking. Components of reflective thinking according to Dewey (1933) is perplexity and inquiry. According to Schon (1991) is the trouble and experiment. Two opinions can be compared. The equality of reflective thinking Dewey (1933) and Schon (1991) presented in Table 1. Implicitly, based on the similarities in the nature of each component, the obtained result of the development of reflective thinking in this article.

Table 1. Development of Reflective Thinking

Dewey (1933)	**Schon (1991)**	**Prayitno et al. (2014)**
Perplexity	*Trouble*	*Perplexity*
Uncertainty about something that is difficult to understand.	Difficulties experienced by someone	Difficulties experienced person to continue the next process; doubts about the answer or solution is found or confusion when someone obtained unexpected results
Inquiry	*Experiment*	*Investigation*
The process of repeatedly information that directs the mind to a certain direction.	Investigations conducted by exploring information to obtain an idea to solve the problem..	An investigation by exploiting existing knowledge to look back information or completion process because of a uncertainty or doubts in obtaining answers

Table 1 shows a comparison of reflective thinking Dewey (1933) and Schon (1991) namely: (1) Trouble partial indicator illustrated also in perplexity, such as someone difficulty in problem solving. Perplexity developed by Dewey is not just in trouble, but rather confirms the existence of doubt or lack of confidence their completion. If students are having trouble, doubt or confusion in solving the problem then it is said the students experienced perplexity; (2) Inquiry can be compared with the experiment, because the inquiry has the same properties as the problem that is causing the effort provide a solution. In the process of looking at the problem, a person can remember what you learned and utilized to solve the problem. The process is known as behavioral changes. In other words, students conduct an investigation by leveraging existing knowledge to look back the completion process due to a lack of confidence or doubt in obtaining answers. Students who experience the process said investigation. Therefore reflective thinking in this article is the thinking process that signed the perplexity and then conducted an investigation till find a solution to the problem (Prayitno et al., 2014).

Related with critical thinking, Pagano & Roselle (2009) states that critical thinking is signed with process of evaluated various relevant information which obtained when the reflection in solving problems. The implicitly "evaluation" are revealed by Pagano & Roselle (2009) is process of selecting some of alternative completion which obtained when reflection so that it can be taken into consideration to make a decision. Fisher (2001) states that critical thinking is signed the

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activity of skilled interpretation and evaluation of the information and statements. Interpretation usually construct some completion and produce alternative. Additionally, the evaluation is process of determining something. Evaluation is signed by selecting the best of some of alternatives (Plymouth University, 2010). Based above definition critical thinking, implicitly there are some components of critical thinking. Components of critical thinking according to Fisher (2001) is interpretation and evaluation. According to Pagano & Roselle (2009) is opinions gathered and evaluation. Implicitly, based on the similarities in the nature of each component, the obtained result of the development of critical thinking in this article.

Table 2. Development of Critical Thinking

Fisher (2001)	**Pagano & Roselle (2009)**	**Prayitno et al. (2014)**
Interpretation	*Opinions gathered*	*The constructive activity*
Construct some solutions and produce some alternative	Produce alternative possibility completion obtained from some of the information collected	Constructing an alternative solution that leads to the answers or construction compare alternative
Evaluation	*Evaluation*	*Evaluation*
Select the best of some alternatives	The process of evaluating some alternatives completion.	evaluate completion alternatives and answers the result by considering the relevant information.

Table 2 shows a comparison of critical thinking Fisher (2001) and Pagano & Roselle (2009), namely: (1) Gathered opinions can be compared with the interpretation, because it has the same properties that produce alternative possibilities completion. The possibility of constructing an alternative solution requires a variety of information that has been collected in the process of reflection. The situation is known as construction (construct); (2) Evaluation of critical thinking Pagano & Roselle (2009) and Fisher (2001) can be compared as in selecting or evaluating an alternative solution or answer. This component signed by evaluated alternative solution or answer based considerations. This component is known evaluation. Therefore critical thinking in this article is thinking process that signed the construct and evaluation alternative completion and the best answer based on various considerations (Prayitno et al., 2014).

Some researchers have review the reflective thinking as process towards critical thinking, among others: reflective thinking is the one tool to develop higher-level thinking (Park & Kastanis, 2009); critical thinking is the result of one's reflection in learning (Asare, 2012); reflective thinking to support critical thinking skills in solving social and political problems (Dawe et al., 2005); reflective thinking increases one's critical thinking and understanding which learned (Park & Kastanis, 2011); reflective thinking the beginning of the process of critical thinking specifically refers to the process of analyzing and making judgments (Colley et al., 2012; Choy & Oo, 2012); and reflective thinking is the key of critical thinking (Colley et al., 2012). In the study, Doerr &

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English (2003) showed the students experienced when using phase shift in thinking reflective thinking and critical thinking so as to produce variations of the model answers. However Doerr & English (2003) did not review how the thinking of students in produce the answer. Whereas Pagano & Roselle (2009) write a study of refraction theoretically and not review in the mathematics education. In the research above have not provided description about how process of reflective thinking continued to critical thinking till produce decisions. Therefore, thinking that is characterized by reflective thinking followed by critical thinking and producing decisions is called *refractive thinking*. This shows that the important components of refractive thinking are reflective thinking, critical thinking and refraction (product).

METHOD

The purpose of this study was to explored and classified the processes of students refractive thinking in solving mathematics problems. Refractive thinking indicated from the process of students solving against instrument task "decision-making". This study employs a qualitative approach, aligned with its characteristics, in which the researcher acts as the primary instrument. In this role, the researcher directly collects data through documentation, field notes, or interviews with the research subjects. The research was carried on students in semester second. For this purpose, the research took the data on student at Universitas Wisnuwardhana Malang and Universitas Negeri Malang. Research subjects not randomly selected, However taken with considered his communication skills so disclosure of the thinking process can be done well. In this study, students were asked to complete task and expresses out loud what he was thinking (think out loud) when solving problem. After students obtain completion, research check the students process completion correct to obtain answers. If student experience reflective thinking and critical thinking in produced decision, then student were included in the group of refractive thinking. Each group is filled by two research subjects. If not obtain the desired subject, then the given the task again to students. The process of selecting subjects performed until a saturation of the data, its meaning that appears the same or remain characteristics of some subjects for each category. The many research subjects for each reflective thinking is 2 subject. Determined 2 subject, with consideration that the method analysis used the constant comparative method (Creswell, 2012). The problem given to the students illustrated Figure 1.

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Local Revenue Offices survey 6 district to find out the level of district dependence on the central government. The dependence of regional on the central government can be measured based contribution the Own-Source Revenue (OSR) to income of province. If the contribution of OSR greater and increased then the district dependence to central government is getting low. The value in table below shows the percentage contribution of OSR to income of province based Natural Resources (NR) for three years.

District / NR	A			B			C			D			E			F		
	Th. 1	Th. 2	Th. 3	Th. 1	Th. 2	Th. 3	Th. 1	Th. 2	Th. 3	Th. 1	Th. 2	Th. 3	Th. 1	Th. 2	Th. 3	Th. 1	Th. 2	Th. 3
livestock	19	9	19	12	24	15	14	22	17	23	14	23	21	15	14	11	16	12
Maritime	18	20	13	9	19	19	12	23	17	24	8	8	19	12	19	18	18	24
Forestry	20	15	19	13	18	18	17	19	15	23	13	11	18	18	10	9	17	27
Plantation	9	11	26	23	17	14	20	22	15	17	16	14	16	24	15	15	10	16
Agriculture	25	14	20	14	13	15	19	15	24	16	24	14	16	18	9	10	16	18
Fishery	12	23	8	19	14	24	7	13	9	15	16	21	24	9	23	23	25	15

The brother task is determine the order of district from the lowest to the highest dependence on the central government! Give an explanation for your answer!

Figure 1. Instruments Task for Subjek

RESULTS AND DISCUSSION

This study used a qualitative approach with the involvement 25 subject. Of the 25 subject, 10 subject grouped into refractive thinking with single strategy; 9 subject are grouped into refractive thinking with dual strategy, and 6 subject are grouped into refractive thinking with multi strategy. In This article the authors describe the single refractive thinking with 1 from 10 subject. The results of the process of refractive thinking are presented in Table 3.

Table 3. Refractive Thinking Distribution

Single strategy	Dual Strategy	Multy Strategy
10	9	6
40%	36%	24%

Characteristics of Refractive Thinking with Single Strategy by Subject 1 (S1)

The process of refractive thinking, begins with perplexity S1 to completion. The behavior is look when S1 was silent for a long time.

> S1: *hmm ...* (silent for long time) *how to do? hmm ... this problem is sorted based dependence of district to central government from the lowest to highest. So, district has percentage contribution of each year always increase it means that a low dependency*

The statement above indicates that S1 experience perplexity to make a completion. Efforts to perplexity, S1 read the problems repeatedly to familiar the problem to be solved. S1 can be show that problem to be solved is sorted the district from lowest to highest dependence. In this case the district has higher contribution each year is district have a low dependency. Conversely, if the

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district experienced decrease of contributions for three years, then district was said a high dependency. Based on this relationship, S1 searched the contribution of Natural Resources of each district per year. The next process, S1 determine average of each district. Completion by average used as strategy to determine order of district. S1 think that the completion with average can be used solved to problem. In the process look for average, S1 describes problem into some parts. S1 completed the first district A, B until the F. The process called analytical process, that is a process describes complex problem into some parts so that parts are then completed. The following completion of average by S1.

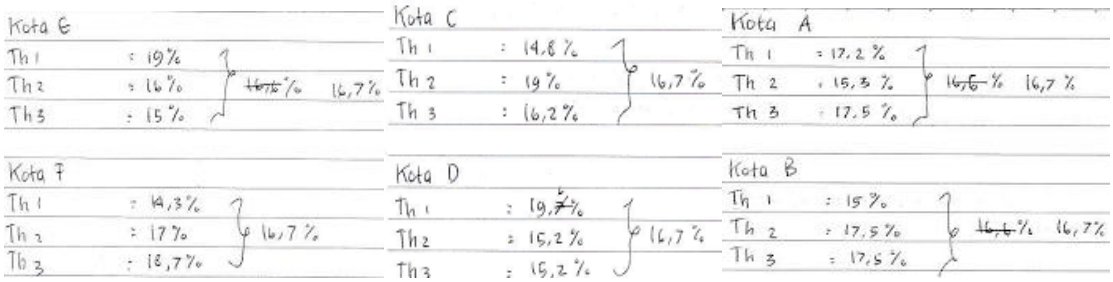

Figure 2. The Results of Subject 1 Obtained by average the Percentages.

The work of S1 begins with sum of percentage all sectors then divided with many of sectors. For example, the first year S1 sum of percentage all sectors and divided with many of sectors, i.e. 6. In the first year obtain average of 17,5%, while the second year average of 15,3% and the third year 17,5%. Based on this average, S1 determine the overall average so obtained 16,6%. This indicates that in determine order of district, S1 completion by average overall the district A. Completion with the average continued until district F. The process completion for other district analog with completion of district A.

Thus overall average of district is the same, i.e. 16,7%. S1 questioned again the average obtained is the same "evidently of average the same?". S1 suspect that the strategy has not been to solved the problem. S1 reading again problems and silent for long time. In this case S1 experienced reflective thinking (Dewey, 1993; Schon, 1991). S1 questioned "if the great contribution and increase then low dependence, how do it?". This show that S1 experience perplexity again when obtained average of the same. S1 think long time again and suspect of criteria "if the greater and increase contribution of Natural Resources then the district dependence to centre of lower".

Based on these criteria, then S1 used another completion with summing the percentage of contribution per year. The process is due to determine the amount of contributions per year. This shows that, when S1 suspect that strategy cannot be used to solve problems, he tried another strategy to solve it. This shows, S1 experience a process of critical thinking (Pagano & Roselle, 2009; Fisher, 2001). S1 summing percentage contribution per year. To explore the thinking process

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of S1 when solved with sum the percentage contributions per year, the research performed interviews. The following interview between research and S1.

R: why do you used such completion this?

S1: Well here mentioned if the great and increase contribution of Natural Resources then the district dependence to centre of lower. Here the many sectors so as to know which one is much summed first all sectors then if it is found each sector summed per year.

S1 claimed that to determine order of the district, the first of summing contribution per year. The completion is based on the criteria of "substantial revenue contribution and increased". To determine the order of the district, the first S1 determine the great of contribution then compared with other district to determine the decrease and increase of contribution every year. The following completion by summed the percentage contribution per year done by S1.

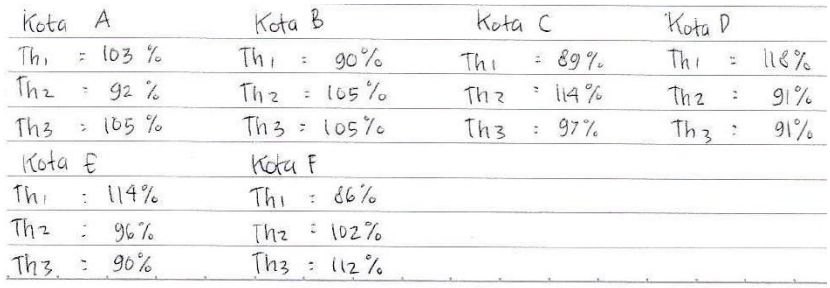

Figure 3. The Results of Subject 1 Obtained by Sum the Percentages.

Based on these answers, S1 summed the contributions of each year. In the first year the contribution amount of district A is 103%, the amount obtained by summed the contribution of livestock till fishery (19 + 18 + 20 + 9 + 25 + 12). In the second year is 92%, which is obtained by summed the contribution of livestock till fishery (9 + 20 + 15 + 11 + 14 + 23) and the third year was 105%, which is obtained by summed the contribution of livestock till fishery (19 + 13 + 19 + 26 + 20 + 8). The process continues until district F. the Completion process in other district analogous to the completion of the district A, namely summed of contribution per year. Based on the results obtained overall, S1 grouping each district. Process doing by S1 is grouping district of each year. S1 again shows the relations criteria "increase" with the amount of contribution each year "the great contribution and increased ..". The statement S1 aware that the criteria "increase" is the keyword to determine the order of the district. Implicitly, S1 can determine relations of the increase and dependency. S1 judging that the district has increased every year is district with low dependence while the district has decreased is an district with a high dependency.

The next process, S1 experience critical thinking with identifying and comparing each district which has increased of contribution each year. Based on the amount of contributions obtained, S1 indicates the lowest dependence is district F. The process done with compared the

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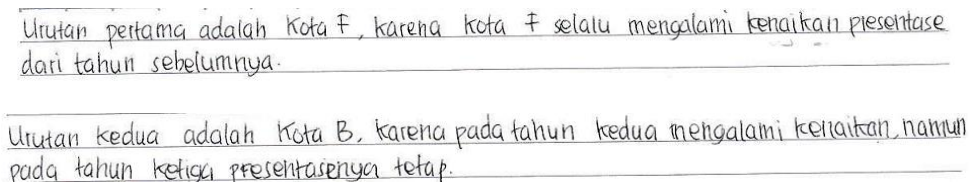
district F and other district. F are considered to have significant increases each year. The next process second order. In the second order, S1 connect again and compared the increase in the amount of the contribution of each district. Based on the amount of contributions obtained, S1 shows second order of lowest dependence of district is the B. if compared with other district, District B has increase in the third year despite constant. The following answers S1 to determine first and second order.


Urutan pertama adalah Kota F, karena kota F selalu mengalami kenaikan presentase dari tahun sebelumnya.

Urutan kedua adalah Kota B, karena pada tahun kedua mengalami kenaikan, namun pada tahun ketiga presentasenya tetap.

Figure 4. The Results of Subject 1 determine first and second order.

The next process determine the third order. In the determine third order, S1 distrustful is district A and C. S1 compared the great of contribution and the increase in district A and C. The district A occur decrease from the first year to the second year, then increase in the third year. The district C occur increase from the first to third year and then decreased in the third year. In the selection for third order, S1 experienced perplexity. S1 think again with to give an alternative completion to indicate the difference between A and C. In the first year of district A is 103%, while district C is 89%, this indicates that the district A was excelled in the first year. While the second year, district A is 92% and C is 114%, this indicates that the district C was excelled. In the third year, district A was excelled as 105%, while district C is 97%, this indicates that the district A has a large amount of contributions for two years, while C is only one year. The following statement by S1 related with determined the third order.

S1: The third order is district A because the first year and third year have greater contribution from the district C and occur increase of contribution from second years.

The following answers to the third order by S1.

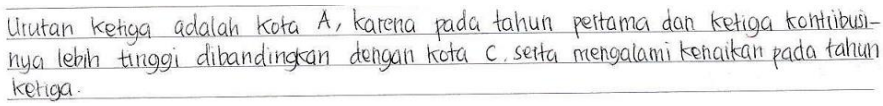

Urutan ketiga adalah Kota A, karena pada tahun pertama dan ketiga kontribusinya lebih tinggi dibandingkan dengan kota C, serta mengalami kenaikan pada tahun ketiga.

Figure 5. The Results of Subject 1 determine third order.

The next process is fourth sequences. S1 put district C as fourth sequences. District C is the comparator A when determining third order, however the district A was excelled compared to district C. Based on these, S1 puts district C as the fourth order. In summed, S1 was connected and compared the increase amount of contribution each district. If district C was compared to other regions (areas D and E), then district C was increased. The following answers to the fourth sequences.

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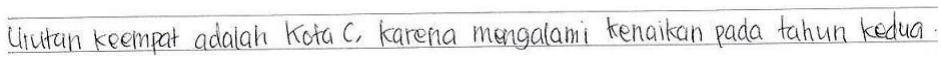

Figure 6. The Results of Subject 1 determine fourth order.

The next process fifth and sixth sequences. In the fifth and sixth sequences , S1 only compared the two district that have not been occupied the previous sequence, i.e. D and E. The District D has decreased but the fixed in the third year. District E has decreased the amount of contribution significantly, that is the first year until third year in a row by 114%, 96% and 90%. District D as district that occupies the fifth order because amount of contribution the same that is second and third year is 91%. This shows that in the second year and third year, district D does not decrease (constant). While in the district E decreased from the first year to third year. This indicates that the district E as an district that sixth order. The following answer in the fifth and sixth sequences.

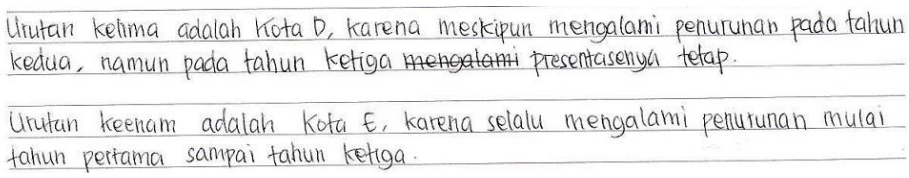

Figure 7. The Results of Subject 1 determine fifth and sixth order.

Based on completion process do by S1 in making decision about district sequence begin lowest to highest of dependence i.e. district F, B, A, C, D , and E. Conclusions are based on the criteria of "amount of great contributions and increased". The following answers district sequence from lowest to highest dependence.

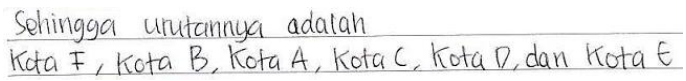

Figure 8. The Results of Subject 1 determine district from lowest to highest dependence.

With these answers, S1 believes the answer. In the process of decision making, a subject need only one completion alternative. Subject only to clarify the criteria contained in the problem as consideration to decide, for example, to identify the contribution of each year. Based on the thinking, S1 experienced refractive thinking with single strategy. The refractive thinking process by S1 can be illustrated in Figure 9.

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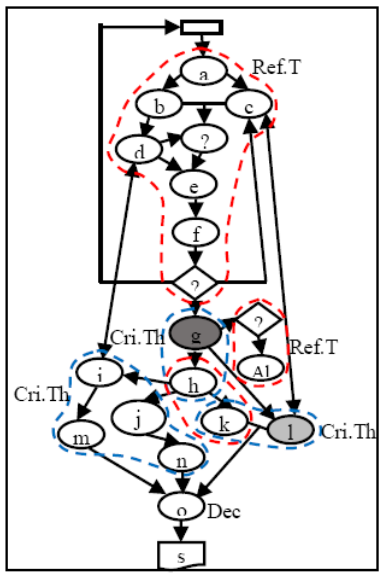

a	:	Sort district based on dependency
b	:	Contribution of NR increased every year
c	:	Lowest of dependency
e	:	The average contribution per year in each district
f	:	The result is the same
g	:	Summed contributions per year
h	:	Summing contributions every district
i	:	The amount of contributions increased
d	:	Remember
Ref.Th	:	Reflective thinking
Dec	:	Decision
Al	:	Other reason
j	:	The amount of contributions decreased
k	:	The amount of contributions dynamic (increase and decrease)
l	:	The amount of contribution increase each year
m	:	The increase in contributions from first year till third year
n	:	The decrease in contributions from first year till third year
o	:	The sort of district: F,B,A,C,D,E
s	:	Finish
?	:	Questioned settlement process
Cri.Th	:	Critical Thinking

Figure 9. The Process of Refractive Thinking With Single Strategy by S1

Based on the description above, it can be described that the scheme of single refractive by students in decision making has been based on Refraction theory (Pagano & Roselle, 2009). Single refractive process occurs when students experience perplexity (trouble). After experiencing perplexity, they make the process of investigation into problem so that produce a strategy summed the contributions of three years. In this case they experience reflective thinking. However, the strategy cannot be used because result obtained the same. They re-investigate to problem so that obtain strategies summed contribution per year. The next process, they experience process of critical thinking namely consider the many contributions each year.

If review from framework by Dewey (1993) and Schon (1991) states that reflective thinking is process occur when someone experience perplexity or trouble thereafter doing investigation to overcome uncertainty, instability and conflicts by students. The produce of decision, the subject only consider some information as comparison, for example considering the total contribution every year. in this case, someone experience critical thinking. If review from framework by Pagano & Roselle (2009) and Fisher (2001) states that critical thinking is process that signed the construct and evaluation alternative completion and the best answer based on various considerations. This process continues until resulted the decision illustrated on Figure 10.

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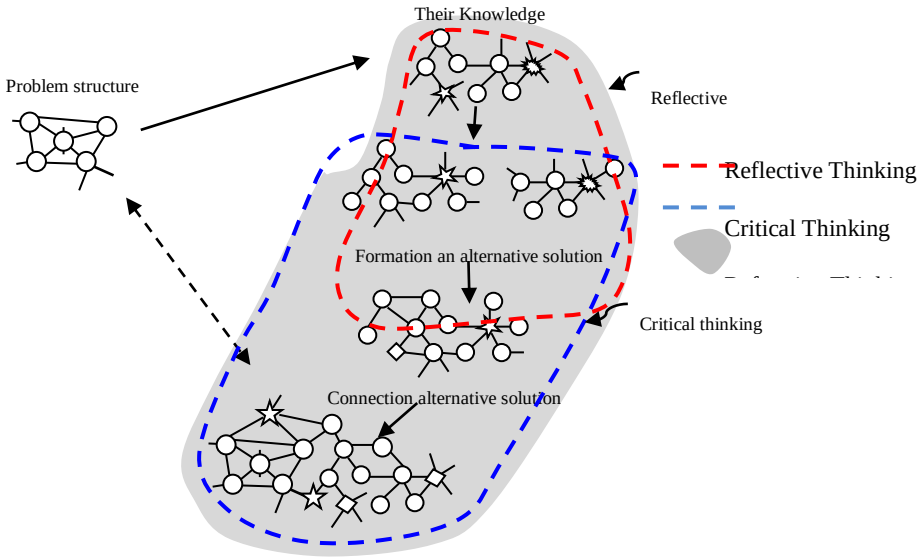

Figure 10. The Process Refractive thinking with single Strategy Based Theory Pagano & Roselle (2009)

CONCLUSION

From the results study that process of refractive thinking with single strategy in solving decision making problems begins when students experience perplexity (trouble). After experiencing perplexity, they make the process of investigation into problem so that produce a strategy summed the contributions of three years. In this case they experience reflective thinking. However, the strategy cannot be used because result obtained the same. They re-investigate to problem so that obtain strategies summed contribution per year or difference the amount of lowest and highest. The next process, they experience process of critical thinking namely consider the many contributions each year. This process continues until resulted the decision. This process is based on an analysis of the thought process experienced by the first subject. However, this conclusion also applies to other subjects who have similar characteristics to the first subject.

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