The Algebraic Thinking Profile of Junior High School Students at Extended Abstract Level of SOLO Taxonomy

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Abstract. This study aims to describe the thinking process of students at the extended abstract level. The descriptive method used in this research is a qualitative approach. The instruments in this study included written test sheets and interview guides. Data analysis techniques used data collection, data reduction, display data, and concluding. The research subjects were two students who had reached the Extended abstract level on the SOLO taxonomy. The results of this study show that students at the extended abstract taxonomy level SOLO can understand the use of variables as generalizations of a number. Still, the majority representation component has not reached the representation indicator by making mathematical modeling in the form of an equation so that they cannot make a hypothesis or prediction in solving the problem related to algebra.

Keywords: Algebraic Thinking, Extended Abstract, SOLO Taxonomy.

Abstrak. Penelitian ini bertujuan untuk mendeskripsikan proses berpikir siswa pada level *Extended Abstract*. Metode deskriptif digunakan dalam penelitian ini dengan pendekatan kualitatif. Instrumen pada penelitian ini meliputi lembar tes tertulis dan pedoman wawancara. Teknik analisis data menggunakan pengumpulan data, reduksi data, penyajian data, dan penarikan kesimpulan. Subyek penelitian merupakan siswa yang telah mencapai level *extended abstract* pada taksonomi SOLO yang diambil secara purposif sebanyak 2 siswa. Hasil dari penelitian ini diketahui bahwa siswa pada level *extended abstract* taksonomi SOLO dapat memahami penggunaan variabel sebagai generalisasi suatu bilangan akan tetapi pada komponen representasi mayoritas belum mencapai indikator representasi dengan membuat pemodelan matematis berupa suatu persamaan sehingga tidak dapat membuat suatu hipotesa atau prediksi dalam penyelesaian masalah yang berkaitan dengan aljabar.

Kata kunci: Berpikir Aljabar; Extended Abstract, Taksonomi SOLO.



INTRODUCTION

Algebra is about abstract structures and about using the principles of those structures in solving problems expressed with symbols (NCTM, 2000). Furthermore, Permatasari and Harta (2018) explain that algebra is a generalization of arithmetic that deals with statements with unknown variables and values to solve problems. In learning algebra, the ability to understand symbols, operations, and rules is required (Andriani, 2015). Meanwhile, in international assessments such as TIMSS, algebra has a proportion of 30% of the total content assessed (Mullis & Martin, 2013). Thus algebra is fundamental for students to learn and understand in supporting other aspects of mathematics and achieving learning goals.

Algebraic thinking consists of generalizations, abstractions, dynamic thinking, modeling, analytical thinking, and organization (Yusrina, 2019). Furthermore, Kieran (Saputro, 2018) defines algebraic thinking as a thought process that involves the development of a way of thinking using algebraic symbols as a tool but not separate from algebra, and also a way of thinking without using algebraic symbols such as analyzing the relationship between quantitative, paying attention to structure, studying change, generalizing, solving problems, modeling, drawing conclusions, and predicting. Based on this description, it can conclude that the ability to think algebraically is a thinking activity that involves processing information, generalizing, making hypotheses, reasoning using mathematical symbols. In order to achieve learning objectives, students are expected to have good algebraic thinking skills. Algebraic thinking skills, namely students in problem-solving, representation, and reasoning in an algebraic context (Yumiati, 2014).

Based on previous studies, it shows that students' algebraic thinking skills do not represent ideal conditions. One of the problems experienced by students in algebraic thinking skills is understanding variables as a representation tool that is very useful for performing generalizations expressions (Zaelani, Warmi, & Ruli, 2019). Meanwhile, Pratiwi and Kurniadi (2018), explain that high school students use variables without a deep understanding of symbol systems' flexibility in algebra. In addition to students 'difficulties regarding symbolic variables and representations, another problem related to students' algebraic thinking skills is the transition process of arithmetic thinking skills to algebraic thinking. Proulx (Sukmawati, 2015), argues that the transition to algebraic thinking is one of the most difficult steps students experience in learning mathematics. The transition process of arithmetic ability to algebra occurred during elementary and junior high schools, because in junior high school students ideally had entered the formal operation stage as suggested by Piaget that at the formal operation stage students already could think abstractly (Ibda, 2015). The diversity of students' algebraic thinking abilities has its level, which is important for teachers and students to pay attention (Maudy, Suryadi, & Mulyana, 2019).

The implication of this causes differences in students' thinking level abilities, which impact the achievement of different learning objectives for each student, so we need a tool to describe a student's level of ability, in this case, algebraic thinking ability. Algebraic thinking is related to the SOLO taxonomy because the SOLO taxonomy can be used to categorize students' algebraic thinking. (Wahyuniar, Shofia, & Rochana, 2018). The SOLO taxonomy (Structure of the Observed Learning Outcome) developed by Biggs and Collis can be used to measure the level of thinking of students in algebraic thinking skills. Following Nuringtyas & Yunianta (2019), SOLO taxonomy in test questions can help to find out how students solve math problems and as a practical evaluation tool to measure the quality of the response to a problem based on the complexity of student understanding or answers to a given problem and is also designed to measure student answers. Biggs and Collis (1982), argue that it is necessary and expected that there is a qualitative assessment in student learning. The assessment can be carried out concerning the structural complexity of the results, and the levels are ordered based on characteristics that include development, increased consistency. The use of organizational principles or relationships, with hypothetical or self-generated principles, are used in the most complex ends.

One of the SOLO taxonomy levels is extended abstract which is a level that has the highest complexity than other SOLO levels. At the extended abstract level, students use a lot of data, then apply the concept, provide interim results, and then link other data or processes to conclude (Widyawati, Afifah, & Resbiantoro, 2017). Furthermore, Christinove & Mampouw (2019), explained that at the extended abstract level students think inductively and deductively. They use two or more pieces of information and connect the information to conclude to build new concepts and apply them. Meanwhile Margayanti (2016) in compiling questions related to the extended abstract level, namely questions that require answers to the results of generalizations, applications outside the context of questions or hypotheses.

Based on the description above, it can be seen that the representation of students' algebraic thinking skills in solving algebraic problems should be a concern as one of the benchmarks in making improvements, planning, and implementing future learning. The problem in this study is how is the algebraic thinking profile of students at the extended-abstract taxonomy level SOLO. This research aims to describe the algebraic thinking profile of students at the extended abstract taxonomy level of SOLO. The algebraic thinking indicator in this study was adapted from the indicator developed by Kamol (2005) in his research which includes variable components and representations. The variable component with the indicator uses the ability to think to understand the use of variables as a generalization of a number and the component representation with the indicator uses the ability to think in solving algebraic problems by interpreting it into a different form of representation. While the achievement indicator for the SOLO extended abstract taxonomy

level in this study was adapted from the indicators developed by Biggs and Catherine (Diana & Irawan, 2017) at the extended abstract level students can apply the generalizations obtained to new and more abstract situations.

METHOD

The approach used in this research is a qualitative approach with a descriptive research type. The data collection area was carried out at SMP Negeri 5 Karawang Barat in class VIII-B. The subjects in this study were selected by 2 students representing the extended abstract level on the SOLO taxonomy after a test and analysis of student responses in answering questions purposively and considering the subject teacher. This study's data collection technique is a test question of algebraic thinking skills that represents the extended abstract level of SOLO taxonomy, unstructured interviews, and documentation.

Data analysis techniques in this study were data analysis techniques according to Miles and Huberman (Sugiyono, 2018) In the analysis of the cumulative data, it is carried out interactively and continues to completion, so that the data is saturated. Activities in data analysis include data collection, the data collected is very large and varied. The data comes from three data sources collected, namely the triangulation data consisting of a written test of algebraic thinking skills, unstructured interviews, and documentation. Data condensation, namely after data is collected from written test results that measure students' algebraic thinking skills. After that data condensation is carried out, namely selecting, focusing, simplifying, determining, and making special notes related to field notes during the study. In this study, the data condensation was carried out by categorizing student responses into the extended abstract taxonomy level SOLO; display data that is after the data is condensed, then the data is presented. It is hoped that the data will be organized, relational, and easier to understand by presenting data. Besides, data that has been collected and condensed, data is presented in the form of narrative or descriptive text which constitutes an interpretation or explanation, and a conclusion drawing or verification, namely after the data is collected, condensed, then presented the data is verified. The initial conclusions put forward are still provisional and will change if no solid evidence is found to support them at the next data collection stage.

RESULTS AND DISCUSSION

Based on the findings in this study, there were difficulties in understanding the variables and the students were unable to make a representation in the form of an equation as a generalization so that in solving the problem they could not get a conclusion or final answer that was relevant to the problems contained in the problem. The following shows the data on the distribution results of the SOLO taxonomy level on the variable components and representations in Table 1.

Tabel 1. Distribution of SOLO Taxonomy Levels on Variable and Representation Components

SOLO Taxonomy Level	Percentage	Total
Unistructural	63,33%	19
Multistructural	36,67%	11
Relational	20,00%	6
Extended Abstract	22,33%	7

Based on Table 1, 22.33% of students reached the extended abstract level in the SOLO taxonomy. Meanwhile, in general, the largest proportion of achievement is at the unstructured level, due to the difficulty of students in understanding variables and not being able to make representations in the form of equations as generalizations. The following shows the responses of students in solving algebraic problems on the variable and representation aspects at the extended abstract taxonomy level SOLO.

Analysis of Student Responses at the SOLO Taxonomy Extended Abstract Level

Variable and Representation Aspects

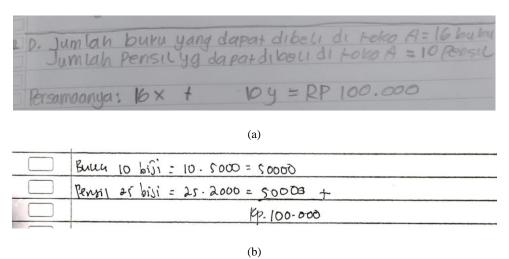


Figure 1. (a) S30 Response and (b) S25 Response

Referring to Figure 1 above shows that S30 and S25 can determine the number of books and pencils that can be purchased for IDR. 100,000.00. The data is obtained through several calculation processes based on the information on the number and price of pencils and books contained in the questions that can be used to determine the price of a book and pencil using the elimination-substitution method. In the previous calculation process S30 and S25, one book was IDR. 5,000.00, and the price of a pencil is IDR. 4,000.00 S30 and S25 can determine a prediction of the maximum amount of books and pencils that can purchase with IDR. 100,000.00. S30 can make mathematical modeling in a two-variable linear equation by writing representation in 16x + 10y = IDR. 100,000.00. That was confirmed in the interview with the S30 as follows.

- R: "How do you predict the maximum amount of books and pencils that can purchase for IDR.100,000.00?"
- 330 : "First look for it right, how much is the price of one book simultaneously a pencil, now I have looked for it in sections B and C."
- R : "How to find it?"
- 330 : "Yes, using the elimination-substitution method can get one book price of IDR. 5,000.00 equals the price of a pencil of IDR. 4,000.00 at store A."
- *R* : "So how do you determine the equation?"
- S30 : "I just guessed it. I tried to calculate if the 16 books got IDR. 80,000.00 means that the pencil must be IDR. 20,000.00 to make it IDR. 100,000.00; So the amount of pencils is 10. I said earlier if x was a book, if y was a pencil, it was 16x + 10y = IDR. 100,000.00."

Based on the passage from the interview with S30, it can assume that S30 can make predictions in determining the number of books and pencils with IDR. 100.000,00 and generalize an equation that models the problem by using a variable representation tool. Shows that the S30 is capable of mineralizing an arithmetic form into algebraic form. Furthermore, Blanton and Kaput (Sharpe, 2019) explain that understanding the arithmetical relations among quantities described in a problem constitutes essential support for the emergence of algebraic thinking. The generalization of the predictions is related to the ability of S30 to apply the knowledge they have and use the data on the questions, both explicitly and implicitly, and applied in solving problems. At the extended abstract level, it can use abstract information from a problem and get new information from a problem to get a result or solution. (Maulidia, Setiani, & Balkist, 2019).

On the other hand, S25 has not been able to determine the representation, namely two-variable linear equation modeling, even though it has obtained the values of x and y in the process of determining the price of a book and a pencil. That requires an understanding of the variables and their applications, not just performing arithmetic operations without looking at the related entities' relationship as an effort to reason algebraically. That was confirmed in the interview with the S25 as follows.

- R: "How do you predict the maximum amount of books and pencils that can purchase for IDR.100,000.00?"
- S25 : "I do not know. I just divided the IDR. 100,000.00 to 50-50 (Each IDR.50,000.00 for books and pencils)."
- R: "Each IDR. 50,000.00? Yes, it is okay, just explain it. You already know what the prices for books and pencils are in store A."
- S25 : "Right in question B, the book price at store A is IDR. 5,000.00, continues IDR. 2,000 pencils, and IDR. 100,000.00 divided by two into 50-50 (Each IDR. 50,000.00 for books and pencils) for the amount book IDR. 50,000.00 divided by IDR. 5,000.00 so there are ten books, for the amount pencil IDR. 50,000.00 divided by IDR. 2,000.00, so there are twenty-five pencils if you add them all up to IDR. 100,000.00."

R: "How do you determine this equation? (Equation of the number of books

and pencils which can be purchased for IDR. 100,000.00)"

S25 : "I do not know."

Based on the interview results above, it can conclude that S25 can predict the number of books and pencils purchased for IDR. 100,000.00. Meanwhile, S25 has not provided a generalization regarding the statement (*maximum amount of books and pencils that can purchase*) through a two-variable linear equation, for example, in the form of 20x + 25y = IDR. 100,000.00. So that S25 has not been able to reach the variable and representation components. Authary & Nazariah (2019) explain that algebraic reasoning is a generalization of arithmetic, which usually uses letters as variables. It is known that S25 in solving the problem makes mathematical modeling, then uses the elimination-substitution technique to determine the book and pencil price. The new data use to predict the maximum amount of books and pencils that can be purchased for IDR. 100,000.00 through mathematical modeling to represent the problem.

There are 23.33% of students who can reach the extended abstract level, the majority can determine the price of one book and pencil, but some have not been able to model the prediction of the maximum amount of books and pencils that can be purchased for IDR. 100,000.00 algebraically but arithmetically as was done by S25, which is without involving variables in representing the problem. At an extended abstract level, students must create a flexible connection among mathematic representation and generalize all mathematical concepts (Afriyani, Sa'dijah, Subanji, & Muksar, 2018). Furthermore, Philipp and Schappelle (Sharpe, 2019) suggest that students experience difficulties in generalizing because they cannot understand variables. Tabach and Friedlander (2017), explain that students difficulties with symbolic expressions in general and with an equivalence of expressions in particular. One frequently mentioned source of difficulties students' students lacks of understanding of symbols or letters.

CONCLUSION

Based on the analysis and discussion results, it can be concluded that the students' ability to think algebraically at the extended abstract level in the variable component can understand the use of variables as a generalization of a number. However, the majority representation component has not been able to solve algebraic problems by interpreting it into a representation form different from making mathematical modeling in the form of an equation to not make a hypothesis or prediction in solving problems related to algebra.

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